GCSE Maths – Algebra

Sketching Graphs – Exponential and Trigonometric (Higher only)

Notes

WORKSHEET

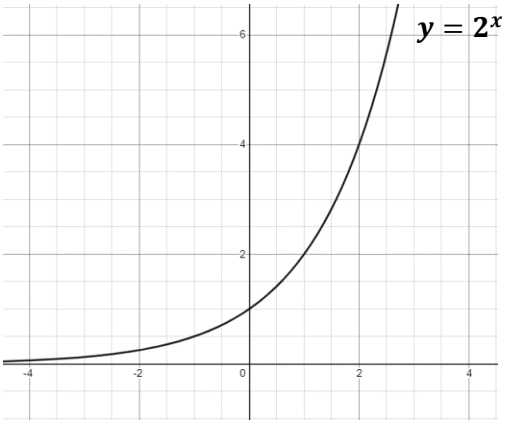
Sketching Graphs – Exponential and Trigonometric (Higher only)

Exponential functions

An exponential function is a function of the form

$$y = k^x$$

with $k > 0$.



As the name would suggest these functions **increase 'exponentially'**. This means they increase at an increasing rate. This can be seen in the graph on the right where the curve gets steeper over time.

Exponential functions are **always positive** since k is positive and a positive number raised to any power is also positive.

- If $k > 1$, the graph will show an increasing curve like the one shown above.
- If $k = 1$, the graph will be constant and equal to 1.
- If $0 < k < 1$ the graph will be a decreasing curve which will get closer and closer to zero over time (but never reaches zero).

To **sketch** an exponential graph, begin by completing a **table of values.** The table displays the corresponding y value for chosen x values, found using the given exponential function.

Example: Sketch the graph of the function $y = 2^x$ between $x = -2$ and $x = 4$

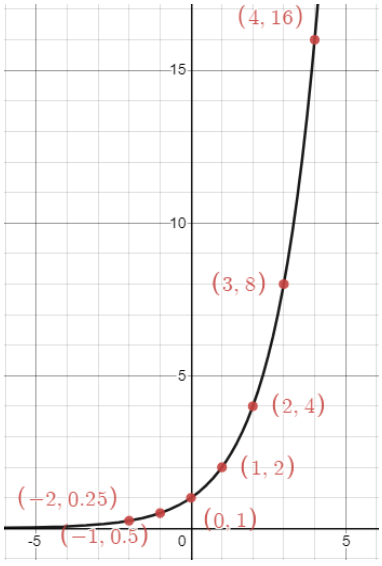
1. *Create the table of values for the values of x.*

x	-2	-1	0	1	2	3	4
$y = 2^x$	0.25	0.5	1	2	4	8	16

2. *Plot the values obtained as coordinates on a graph.*

   From the table above, we plot the coordinates

   $(-2, 0.25)$
   $(-1, 0.5)$
   $(0, 1)$
   $(1, 2)$
   $(2, 4)$
   $(3, 8)$
   $(4, 16)$.



   Then draw a smooth curve connecting these points.

Trigonometric Functions

The trigonometric functions are functions that take angles as their inputs and relate them to the ratio of the sides of a right-angled triangle. **Sin**, **cosine** and **tan** are trigonometric functions.

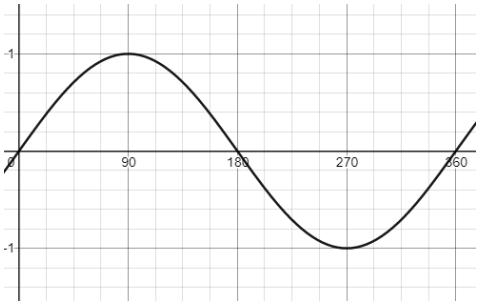
Trigonometric functions are **periodic**. This means they **repeat** their values after regular intervals. The length of the interval which repeats is called the **period**.

- Sine and cosine both have a period of 360°.
- Tan has a period of 180°.

When **sketching trigonometric graphs**, remember that the shape of the curve repeats infinitely in both directions. It is easy to sketch the trigonometric curves, as long as you remember the **identifying features** of each graph.

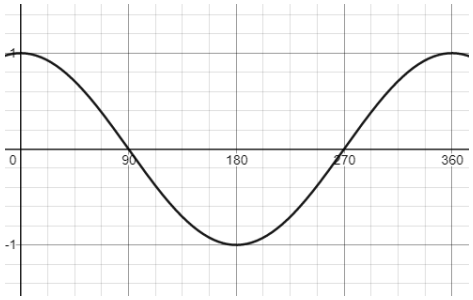
Sine

- $\sin x = 0$ at $x = 0^\circ$, 180° and 360°
- $\sin x = 1$ at $x = 90^\circ$
- $\sin x = -1$ at $x = 270^\circ$
- Repeats every 360°



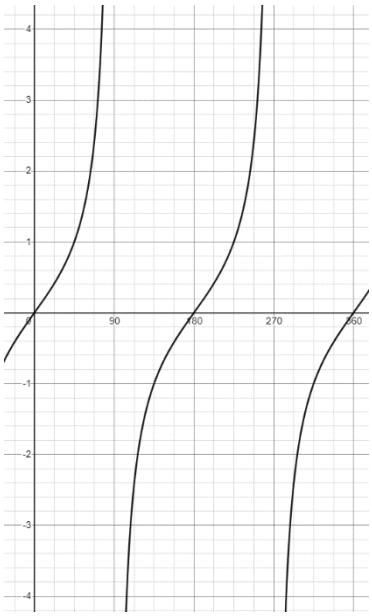
Cosine

- $\cos x = 0$ at $x = 90^\circ$ and $x = 270^\circ$
- $\cos x = 1$ at $x = 0^\circ$ and $x = 360^\circ$
- $\cos x = -1$ at $x = 180^\circ$
- Repeats every 360°



Tan

- $\tan x = 0$ at $x = 0^\circ$, 180° and 360°
- $\tan x$ approaches infinity at odd multiples of 90° and then 'starts again' from minus infinity.
- Repeats every 180°



Exponential and Trigonometric Graphs (Higher) – Practice Questions

1. Sketch $y = 3^x$ between $x = 0$ and $x = 3$.

2. Sketch $y = \left(\frac{2}{3}\right)^x$ between $x = -7$ and $x = 1$.

3. Sketch $y = \tan x$ between $x = -270°$ and $x = 270°$.

4. Sketch $y = \cos(x + 90°)$ between $x = -180°$ and $x = 180°$.

5. Sketch $y = -2\sin x + 1$ between $x = -90°$ and $x = 180°$.

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.